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## CALCULUS OF A UNITARY FUNCTION IN REALITY

Prof. Jay prakash Tiwari  
Department of Mathematics,  
Patel group of institutions, indore

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### Abstract

: In the paradigm of orthogonal calculus, we build a calculus of functors to examine 'determines the following with reality,' such as the Real categorizing  $BU_{\mathbb{R}}(-)$ , space functor. The calculus creates a Taylor tower, with the n-th tier classified by a  $C_2 \rtimes U(n)$  action on the spectrum  $(n)$ . We also explore model category consideration, resulting in a zigzag of Quillen equivalences between spectra with just an operation of  $C_2 \rtimes U(n)$  as well as a system models just on categories of inputs functors that encapsulates the homotopy theory of the Taylor tower's n-th layer.

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**1. Introduction**  $J$ -spaces are systematically studied using the orthogonal and unitary calculi [27, 24], when  $J$  is the subcategory of finite-dimensional real initial feature space or complicated internal product spaces, accordingly. The essential notion of such calculi is to estimate a given functor with a series of polynomials functors, akin to Taylor's series in mathematical analysis. What could we say when the  $J$ -spaces arrive is a natural thing to ask symmetry in the form of collective action? This equivariance is important in Goodwillie calculus. Dotto [5, 6, 7] and Dotto and Moi [8] have looked at it. This is the first step in the much broader process. The accompanying math with reality is a quest to comprehend partition orthogonal calculus. This is unitary calculus, with the  $C_2$ -action on the domain of complicated internal product spaces generated by complex conjugation taken into consideration. Researchers demonstrated the significant resemblance between orthogonal and unitary calculi and real and complex topological K-theory in [24, 25]. In fact, the observations of [25] were motivated by this analogy. Due to Atiyah [2], the mathematics with reality addressed here fits within this comparison, taking the role of K-theory with reality, thus the designation 'with reality.' The concept is similar to that of orthogonal and unitary calculus, but modified to allow for the exact correct balance that comes from complicated conjugation. See Proposition 3.13 for a definition of a polynomials functor having reality and a construction of a polynomial approximations functor that is given as the fibrant substitution in an appropriate conceptual framework on the subcategory of functors having reality. We create a structural model on the categories of functors with reality that encapsulates the homotopy theory of  $n$ -homogeneous functors, in especially the  $n$ -th tier of the Taylor skyscraper, using localization techniques (see Proposition 4.2). In 2.1, and 3.1, we obtain a zigzag of Quillen equivalences, allowing one to classify  $n$ -homogeneous functors as (orthogonal) spectrum including an activity of  $C_2 \rtimes U(n)$ . The final outcome is a Taylor tower in which these wavelengths with a  $C_2 \rtimes U(n)$  action classify the  $n$ -th layer  $(n)$ . The scenario differs substantially from that of orthogonal and unitary calculus; having an extra step in the zigzag of Quillen equivalences is convenient. This additional step demonstrates that the calculus is sensitive to a insertion of extra equivariance. It will contribute to much more complex computations, such as the addition of [1] to the evaluation of the  $BU_{\mathbb{R}}(-)$ , is the real categorizing space of the unitary group functor. Oman and Barne [3] built a zigzag of Quillen equivocations inside the orthogonal calculus utilising only one intermediate stage among the  $n$ -homogeneous equivalences. Considering an act of  $O$ , model structure and orthogonal spectra  $(n)$ . The unitary calculus is based on the. The author [24] originally presented a connection between the Quillenequivalency and the unitary  $n$ -homogeneous structural model was then provided for unitary spectrum with just an action of  $U(n)$  between unitary spectra with such a  $U(n)$  action and orthogonal harmonics with an  $O(n)$  action  $U(n)$ . This additional step could be incorporated into to the Quillen equivalence between the unitary and the times tables. [24], Theorem 3.1], intermediary categories

and unitary spectra with an action of  $U(n)$ , to the left (resp. right) Quillen adverbial adverbial adverb compose in order to provide to the left (resp. right) Quillen adverbial adverb.. None of the Quillen equivalences can be constructed to minimize the duration of the zigzag in the 'using reality' scenario as it would require combining left Quillen functors with right Quillen functors, which are neither left nor right Quillen in general. As just an intriguing digression, we additionally enhance the notion of calculus with reality, analogous to Atiyah's KR-theory, by establishing a category equivalency between both the category  $C_2 \otimes \mathcal{E}_1^R$  in our zigzag and Schwede's Real spectrum (see Definition 3.1), see Proposition 3.2. When we combine this equivalence of categories with our zigzag of Quillen equivalences, we can conclude that the stochastic theory of  $n$ -homogeneous functors with reality is identical to the homotopy theory of Actual spectrum with one element.

**Main outcomes and organization:** We establish the Real version of Weiss' Stiefel combinatorics [27, Section 1] in Section 2. The Real Stiefel combinatorial optimization are critical to understanding creating the derivatives of a functor and the homotopy theory of polynomial functors with reality. In specifically, we check Proposition 2.1's 'crucial' consequence again for validity of the calculus.

**PROPOSITION A.** For all  $V, W \in \mathcal{J}^R$ , as well as  $n \geq 0$  the bundle of spheres  $S\gamma_{n+1}^R(w, v)$  is homeomorphic of  $\text{Chocolim}_{0 \neq U < C^{n+1}} \mathcal{J}^R(U \otimes w, v)$ .

In Section 3, researchers describe polynomial functors as well as the polynomial approximations functor, as well as a modeling categories that embodies the homotopy theory of  $n$ -polynomial functors. This projective prediction model on the subcategory of functors with reality is supplied as a left Bousfield localisation of this model structure. In Section 3, we look at  $n$ -homogeneous functors, with the  $n$ -th layer of both the Taylor tower as an illustration. All of those are  $n$ -polynomial functors which have trivial  $n$ -polynomial approximations. In addition, we present a structural model for the categories of functors with reality that encapsulates the homotopy theory of  $n$ -homogeneous functors. A right Bousfield localisation of the  $n$ -polynomial structural model is it this model structure. The strata of the Taylor tower are cofibrant-fibrant in the  $n$ -homogeneous structural model, resulting in the following Taylor tower. In Section 5, researchers construct a functor's derivatives and prove that an  $n$ -polynomial functor's  $(n + 1)$ -st derivative with reality is trivial. On the  $n$ -th level intermediate category, we establish the intermediate categories, which are the natural home for the derivatives, and produce a stable model structure.

In Section 6, we demonstrate a zigzag of Quillen equivalences between the intermediate category and the category of spectra with an action of  $C_2 \otimes U(n)$ , which is two of the three phases in our zigzag of Quillen equivalences ( $n$ ).

**THEOREM B:** There is a pair of adjacent pairs.

$$\mathcal{E}_n : C_2 \otimes U(n) \mathcal{E}_1^R \Leftrightarrow C_2 \otimes \mathcal{E}_1^R[U(n)] : \mathcal{E}_n$$

When both categories have their stable modeling structures, this is a Quillen equivalence.

**THEOREM C.** There may be a pair of adjacent pairs.

$$AL_\mu : \mathcal{S}_L^U[C_2 \otimes U(n)] \Leftrightarrow C_2 \otimes \mathcal{E}_1^R[U(n)] : \mu,$$

When both category have their stable modeling frameworks, this is a Quillen equivalence. Researchers also prove a categorical equivalence in between Schwede categories of Real spectra (Definition 3.1) and  $C_2 \otimes \mathcal{E}_1^R$  (Proposition 3.2).

**PROPOSITION D.** The grade  $C_2 \otimes \mathcal{S}_L^U$  (Real Spectra) is similar towards the category  $C_2 \otimes \mathcal{E}_1^R$  (Extended Real Spectra).

As part of a Quillen equivalence between both the  $n$ -homogeneous structural model just on categories of functors to reality as well as the  $n$ -stable system models on the  $n$ -th level intermediary category, we prove – as Theorem 7.8 – that the differentiating functor is a right Quillen functor in Section 7.

**Notation and conventions.** We establish an isometric isomorphism  $C^n \cong C_2 \otimes R^n$  for all time. This isomorphism limits the complicated conjugate on  $C_n$  to that of the source  $C_2 \otimes R^n$  the semi-direct product of a group of two members is denoted by  $C_2 \otimes U(n)$  the degree  $n$  unitary group Researchers use  $g$  to signify the non-identity element of  $C_2$ .

**2.Real Stiefel combinatorics.** These concepts of derivatives, as well as, as well as polynomials are central here to calculus theory. These derivative in unitary and orthogonal are built using relationships between specific index various segmentation as  $n$ -th jet categories (see [27, Section 1] and [24, Definition 3.1]). All adjunctions utilised in the Quillen equivocations of [3] and [24] are defined by the interplay between such categories. While dealing with mathematics of truth, this task becomes slightly more complex. To fix a

complicated conjugation have a well theory, we must properly select our indexing category. We begin by generating the n-th jet category and showing links between them in such a world.

**2.1. The universe.** Since universe C is indexed in unified calculus, hence in [24], researchers looked at functors which accept value on trivially small inner product subcarriers of C. We need the universe with all of its finite-dimensional subspaces to remain close by complicated conjugation in a calculus without reality setting, because with it, we wouldn't be allowed to apply a C2 Top\*-enrichment to the subcategory of inputs functors. C is incorrect in this case,

**2.2. The indexing categories:** The building of the universe begins after the proper universe is in place.

The category of indexing are real that follow the orthogonal and unitary calculus versions [27, 24].

**3. Polynomial functors with reality:** These concepts of polynomials, as well as derivatives, as well as derivatives are central here to calculus theory. These derivative in orthogonal and unitary calculus are built using relationships between specific index various segmentation as n-th jet categories (see [27, Section 1] and [24, Definition 3.2]). All adjunctions utilized in the Quillen equivocations of [3] and [24] are defined by the interplay between such categories. While dealing with mathematics of truth, this task becomes slightly more complex. To fix a complicated conjugation have a well theory, we must properly select our indexing category. We begin by generating the n-th jet category and showing links between them in such a world.

**DEFINITION 3.1.** From  $J_0^R$  to C2 Top\*, define  $C2 \otimes \mathcal{E}_1^R$  as the category of C2 Top\*-enriched functors.

The notation was chosen with care  $C2 \otimes \mathcal{E}_1^R$  can be thought of as an input category for unitary calculus with an interwoven C2-action. There are various levelwise modeling architectures in this area. The model structure is determined by the model chosen. C2 Top\* has a structure. We decided to use the Quillen structural model that was copied from the addition\* at the top.

$$C2+^{\wedge} \text{Top}^* \Leftrightarrow C2\text{Top}^*:i^*$$

Where,  $i$  is the trivial group's inclusion in C2. The fibrations are the fundamental Serrefibrations, and the weakly equivalences are an under insufficient homotopy equivalences. The generating cofibrations are of the form  $C2+^{\wedge} i$  for  $i \in I$  and  $C2+^{\wedge} j$  for  $j \in J$ , where I and J indicate the collection of generating cofibrations and generating acyclic cofibrations for the Quillen structural model on Top\*, accordingly. Cells of the form  $(C2)+^{\wedge} D_+^n$  are used to construct the C2-CW-complexes. As a result, on  $C2 \otimes \mathcal{E}_1^R$ , we utilise the model structure below.

**PROPOSITION 3.2.:** Mostly on category  $C2 \otimes \mathcal{E}_1^R$ , there may be a cellular, appropriate, and topological model structure, in which a map  $f: S \rightarrow E$  is a weak equivalence (resp. fibration) if and only if for each  $U \in J^R$ ,

$f(U): S(U) \rightarrow E(U)$  is a weakly homotopy equivalence (resp. Serrefibration) in C2 Top\*. The generation (acyclic) cofibrations of the Quillen model structure on Top\* are of the type  $J_0^R(U, -)^{\wedge} C2+^{\wedge} i$  for  $i$  a generating (acyclic) cofibration. The polynomial model structure is such rationale for the this decision. Designers wouldn't be able to demonstrate that polynomials approximation functors maintain levels weak equivalences on constants if we started with the fixed-points structural model on C2 Top\* because this relies on the fact that homotopy colimits maintain weaker equivalences. The polynomial approximations functors just wouldn't interact well to the fixed-points structural model on  $C2 \otimes \mathcal{E}_1^R$  because homotopy colimits need not communicate for point sets in general. As a result, while employing the fixed-points model structure, we were unable to demonstrate the presence of a Bousfield-Friedlander local structural model as in [3] and [24, Proposition 3.3].

**PROPOSITION 3.2..** Tynan already studied the theory of a calculus containing reality in his thesis [26]. Tynan examined functors from the category of real embedding spaces, however the complexification functor is usually used to expand towards the domain of real inner product spaces. This is what we're talking about: complexified genuine kernel spaces [30, 31]. We believe because so many of the functors one would want to explore in a calculus of reality originate from unitary calculus rather than orthogonal calculus, this method is much more natural. It ought to it should be noticed that the source category of Tynan and the output category of each category of input Furthermore [30, 31, 32, 33], the authors benefit from his usage of the complexification functor previously published research comparing orthogonal and unitary calculi [25]. We also favour our approach since it clarifies the equivariance and classifies the n-homogeneous functors [29, 30, 31, 32, 33], which is something missing from [26]. We obtain the follows as a direct implication of the existence of the projective structural model:

**3.2. Reality and polynomial functors:** Polynomial functors are  $C2 \otimes \mathcal{E}_1^R$  objects that satisfy additional requirements that cause them to act like polynomial functions in differential calculus. Let's begin with a definition. [27, Definition 3.5] and [24, Definition 2.1] are examples of this.

**DEFINITION 3.5.** Let  $n \geq 0$ . If the canonical map is polynomial of degree less than or equal to  $n$ , a functor  $E \in C2 \otimes \mathcal{E}_1^R$  is polynomial of degree less than or equal to  $n$  or  $n$ -polynomial.

**REMARK 3.6.** The homotopy limit is built to account for the fact that  $R_{n+1}$  is a category object in the category of spaces (see [27] and [19, Appendix]). The homotopy limit, in instance, is the totalisation of a cosimplicial space and can thus be stated as an enriched end. It gets a  $C2$ -action from the general rule that if a diagram has a  $G$ -action, the end inherits it via the induced map on equalisers:

**REMARK. 3.6.** The homotopy limit is built to allow for the fact that  $R_{n+1}$  are a categories item in the categories of space (see [27] and [19]). This homotopy bound is the tantalization of a simplicial space in specific, whereas both a result, an enriched conclusion can be conveyed. It gains a  $C2$ -action as a result of the general fact that if a diagram has a  $G$ -action, the ending inherits it through the induced effecta map of compensators. Whenever a functor is  $n$ -polynomial, there seems to be an alternate characterisation that is required to characterise the fibrant items in the  $n$ -polynomial structural model. After observing the  $C2$ -equivariance of Proposition 3.2, the proof proceeds as in [27, Proposition 3.3].

**PROPOSITION 3.7.** Let  $E \in C2 \otimes \mathcal{E}_1^R$  be the value. If and only if,  $F$  is  $n$ -polynomial.

$P^*: E(V) \rightarrow C2 \otimes \mathcal{E}_1^R (S\beta_{n+1}^R(V, -) + E)$ ,

For all  $V \in J_0^R$ , there is weak homotopy equivalence.

We'll now move on to discussing a functor's polynomial approximations. This definition is quite similar to those found in orthogonal and unitary calculus [3, Definition 3.5] and [24, Definition 3.5].

**3.3. The structure of the  $n$ -polynomial model.** The  $n$ -polynomial model structure is a major feature of Barnes and Oman's work [3]. The  $n$ -polynomial approximation functor is a fibrant replacement functor in this model structure, which captures the homotopy theory of  $n$ -polynomial functors – these are the fibrant objects. The  $n$ -polynomial model structure for calculus with reality derives from the orthogonal and unitary counterparts [3, Proposition 3.3 ] and [24, Proposition 3.2] because we are employing the underlying model structure on  $C2 \otimes \mathcal{E}_1^R$ .

**PROPOSITION 3.3.** On the category  $C2 \otimes \mathcal{E}_1^R$ , there is indeed a cellular, appropriate, and topological model structure containing weak equivocations all maps  $f: S \rightarrow E f$ , and as such the  $Tnf: TnS \rightarrow TnE$  is a weak equivalency in the underpinning structural model on  $C2 \otimes \mathcal{E}_1^R$ . The fibrations are indeed the levels fibrations

$f: S \rightarrow E$  in  $C2 \otimes \mathcal{E}_1^R$  and the squared fibrations is a homotopy reversal. This model structure's cofibrations are the cofibrations of the underlying model structure on is a homotopy reversal. This model structure's cofibrations are the cofibrations of the underlying model structure on  $C2 \otimes \mathcal{E}_1^R$ . The  $n$ -polynomial model structure is denoted by the letters  $n$ -poly- $C2 \otimes \mathcal{E}_1^R$ . The  $n$ -polynomial model structure is denoted by the letters  $n$ -poly- $C2 \otimes \mathcal{E}_1^R$ .

**Conclusion:** Those that are familiar with both the orthogonal as well as unified calculus will recognize the grand principle of this calculus. This section gathers a number of instances for the reader, so those who are acquainted with both the theory's general concept could see this novel calculus in action then reference back to the appropriate sections as needed. The calculus of reality and unitary calculus are very similar. This is not surprising; given unitary calculus is the calculus that results from 'forgetting' the  $C2$ -action that we have incorporated into the calculus with reality. The parameterized functors are of special importance to us. They were vital in comprehending convergence results in orthogonal and unified calculus , and they work well with the comparison functors. The representable functors' derivatives with reality are now described using the model categories defined in this study. Consider the  $J_N^R(0, -) = nS$  functor. Then we'll apply this to  $J_N^R(V, -)$  for all  $V \in J_N^R$  and all  $n \geq 0$ .

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